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Vibro-acoustics Response of a Rectangular Thin Plate Under Different Force Location and Different Thickness

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Abstract

Analytical dynamic response and sound power radiation of a rectangular simply supported thin plate under external excitation force and different force location and thickness are studied in this paper. The natural frequencies and vibration response of the plates are obtained by means of the Rayleigh–Ritz method. Sound radiation characteristics of the rectangular plate are derived using Ray-leigh integral formula. The non-dimensional frequency parameters of the rectangular plates with simply support boundary condition are presented in the paper, which may be useful for future researchers. Accuracy of natural frequencies is confirmed with pervious researches. Finally, the effect of point force location and thickness, on acoustic characteristic of simply supported recangular plate are studied in this paper. The relationship of sound power level and point force location also plate thickness is deducted. Meanwhile, in this paper , some interesting points are found when analysing acoustic radiation characteristics of simply supported recagular plates.

Keywords: vibro-acoustics response; rectangular thin plate; sound power radiation.

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1. Introduction

Rectangular plates are one of the most common structures encountered in many branches of technology, such as mechanical, automotive, aerospace, nuclear, marine and chemical engineering, on the other hand, noise and vibration control is one of the most important research areas in engineering such as vehicle noise. So, on order to low-noise machinery design, it is meaningful to study the dynamic and acoustic characteristic of rectangular plates.

Du et al. [1] studied dynamic and Sound radiation characteristics of rectangular plates with various combinations boundary conditions in the Rayleigh-Ritz procedure. Their study has identified the stiffness values of translational springs have greater effect on natural frequency of plate than rotational springs. With the increase of aspect ratios, the natural frequency increases. Natural frequency of thin plate varies greatly with different boundary conditions. In the combination of clamped boundary condition and elastic boundary condition, with the increase region of clamped boundary condition the frequency parameter increases. Qiao et al. [2] conducted sound characteristic radiated from un-baffled rectangular plates with clamped support and elastically support. They showed that the sound power level radiated from un-baffled rectangular plates is lower than the one radiated from baffled rectangular plates and the vibration energy of them is a little different. Because of the acoustic surface pressure jump on the plate surface, the radiation sound efficiency of un-baffled rectangular plates is lower than baffled rectangular plates but there is little difference in low frequency under 200 Hz. Kong et al. [3] analysed the coupled vibro-acoustic modelling of tensioned membrane backed by the rectangular cavity based on the analytical method. In their study, an accurate and computationally efficient program is generated based on the Hamilton's principle, and the Rayleigh method is used to derive the sound radiation power and sound transmission loss of the coupled model. The free and forced response of the coupled model have been computed by FEM to validate the accuracy of the proposed method. Compared with FEM, their method only needs a few elements to get the calculation results of the coupled model, and it takes much less time. In fact, the method is an analytical or semi-analytical method, which does not depend on grids and requires only a few degrees of freedom to compute the whole frequency range, so the computational efficiency is very high. Gunasekaran et al. [4] studied the buckling and vibroacoustic representations of a simply supported plate under non-uniform edge loading based on the analytical method. Where the strain energy approach is adopted to estimate the buckling load. In their study, free and forced vibration response of the plate is obtained using an analytical method based on Reddy's third-order shear deformation theorem while sound radiation behaviour is analysed using Rayleigh Integral. They found that the buckling load parameter significantly influenced by non-uniform edge load variation. Singh et al. [5] presented an analytical investigation of the sound radiation behaviour of a thin exponential functionally graded material plate using the classical plate theory and Rayleigh Integral with the elemental radiator approach. In their study, it has been found that, for the considered plate, the modulus ratio significantly influences sound power level and sound radiation efficiency. Effects of modulus ratios on the sound power level showed frequency shift over stiffness control region in the low-frequency range (first mode for all modulus ratios). The different values of damping loss factors do not significantly influence radiation efficiency for the given material constituents of the functionally graded plate. However, the selection of material constituents influences the radiation efficiency peak. Yonggan Sun [6] conducted vibration and acoustic radiation of stiffened plates in the presence of in-plane normal and shear loads using the finite element method. In structural modelling, the plate and stiffeners are treated as separate elements and the strain and kinetic energies of the stiffened plate with an elastic boundary are introduced. The results show good agreement with those obtained using other methods. Parametric studies show that in-plane normal forces have obvious influences on the acoustic radiation efficiency and the sound power level of the structure. Furthermore, the position of in-plane normal forces warrants attention; e.g., the farther the boundary in-plane normal forces from the boundary constraint are, the greater the effect on the acoustic performance is. However, in-plane edge shear loading has little influence on the acoustic performance of structures. Zhang et al [7] developed a plate-cavity system to study the vibro-acoustic coupling characteristics based on an improved Fourier series method (IFSM). The established coupling system consists of an acoustic cavity with rigid walls or impendence walls and a single or double thin laminated rectangular plate with various elastic boundary conditions. The results obtained by developed method show good convergence and agreement by being compared with those of the literatures or finite element method (FEM). The natural characteristics of the plate-cavity coupling system are studied. The structure vibration response and acoustic pressure response are also investigated by applying structural force to the plate and applying the monopole source into the acoustic cavity.

The researches mentioned above are mainly focused on free vibration, forced vibration and sound radiation of plates and shells with different boundary conditions. In present study, researchers focus on different point force location and different thickness of a simply supported rectangular plate.

2. Mathematical modelling

Consider a rectangular plate of length a in direction x and length b in direction y, as shown in figure 1, in an infinite rigid baffle. The equation of motion of rectangular plate under a variable, time-dependent, transverse load p(x, y, t) is given by Eq. 1.

$$D\nabla^2\nabla^2 w(x,y,t) = p(x,y,t) - \rho h \frac{\partial^2 w}{\partial t^2}(x,y,t) \tag{1}$$

A solution of the above nonhomogeneous, partial differential equation must satisfy the prescribed boundary and initial conditions. An exact solution can be obtained by using the following procedure. First, let us solve the problem of free vibrations of a plate, and determine the natural frequencies ω_mn and the corresponding mode shapes W_mn. natural vibrations are functions of the material properties and the plate geometry only, and are inherent properties of the elastic plate, independent of any load. Thus, for natural or free vibrations, p(x,y,t) is set equal to zero, and Eq. (1) becomes:

$$D\nabla^2\nabla^2 w(x,y,t) + \rho h \frac{\partial^2 w}{\partial t^2}(x,y,t) = 0 \tag{2}$$

Deflection w must satisfy the boundary conditions at the plate edge and the following initial conditions:

$$when \quad t = 0: \quad w = w(x,y), \quad \frac{\partial w}{\partial t} = v_0(x,y) \tag{3}$$

Let us describe a general analytical method (the Fourier method) for determining the natural frequencies of a freely vibrating plate. To solve Eq. (2) and obtain w(x,y,t) in general, one can assume the following solution:

$$w(x,y,t) = (A\cos\omega t + B\sin\omega t)W(x,y) \tag{4}$$

which is a separable solution of the shape function W(x,y) describing the modes of the vibration and some harmonic function of a time; ω is the natural frequency of the plate vibration which is related to vibration period T by the relationship ω=2π/T. Introducing Eq. (9.5) into Eq. (9.3), we have:

$$D\nabla^2\nabla^2 w(x,y) - \omega^2 \rho h W = 0 \tag{5}$$

The lowest frequency is called the frequency of the fundamental mode or the fundamental natural frequency and all other frequencies are called the frequencies of higher harmonics, or overtones. For each frequency ω_mn, there is a corresponding shape function W_mn (x,y) that, on the basis of the homogeneous equations, is determined by a constant multiplier (which can be assumed as being equal to unity). In the case of a rectangular, simply supported plate, the shape function may be taken as:

$$W(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn} \sin\frac{m\pi x}{a} \sin\frac{m\pi x}{a} \tag{6}$$

where a and b are the plate dimensions and C_mn is the vibration amplitude for each value of m and n. Substitution of Eq. (6) into Eq. (5) results in the homogeneous algebraic equation

$$\frac{m^4\pi^4}{a^4} + 2\frac{m^2\pi^2}{a^2}\frac{n^2\pi^2}{b^2} + \frac{n^4\pi^4}{b^4} - \frac{\omega^2\rho h}{D} = 0 \tag{7}$$

Solving this equation for ω gives the natural frequencies

$$\omega_{mn} = \pi^2(\frac{m^2}{a^2} + \frac{n^2}{b^2})\sqrt{\frac{D}{\rho h}} \tag{8}$$

Then, let us introduce a load p(x, y, t) in the form of series extended in eigenfunctions (the mode shapes), i.e.

$$p = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} f_{mn}(t) W_{mn}(x,y) \tag{9}$$

We seek a solution of Eq. (9) in the form

$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} F_{mn}(t) W_{mn}(x,y) \tag{10}$$

The following equation takes place for the function F_mn:

$$\ddot{F}_{mn} + \omega^2{}_{mn} F_{mn} = \frac{1}{\rho h} f_{mn}(t) \tag{11}$$

$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} [A_{mn} \cos \omega_{mn} t \ + \ B_{mn} \sin \omega_{mn} t \ + F^{(p)}{}_{mn}(t)] W_{mn}(x,y) \tag{12}$$

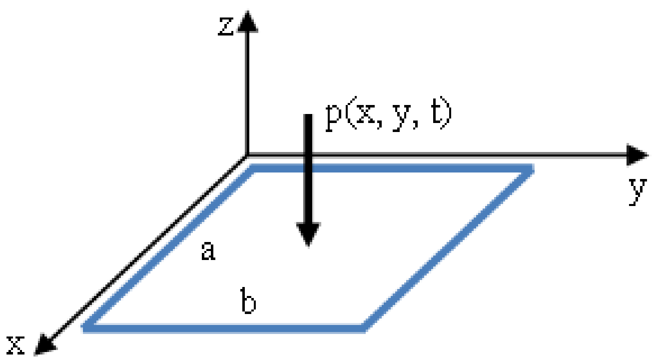

Figure 1. Rectangular plate under force

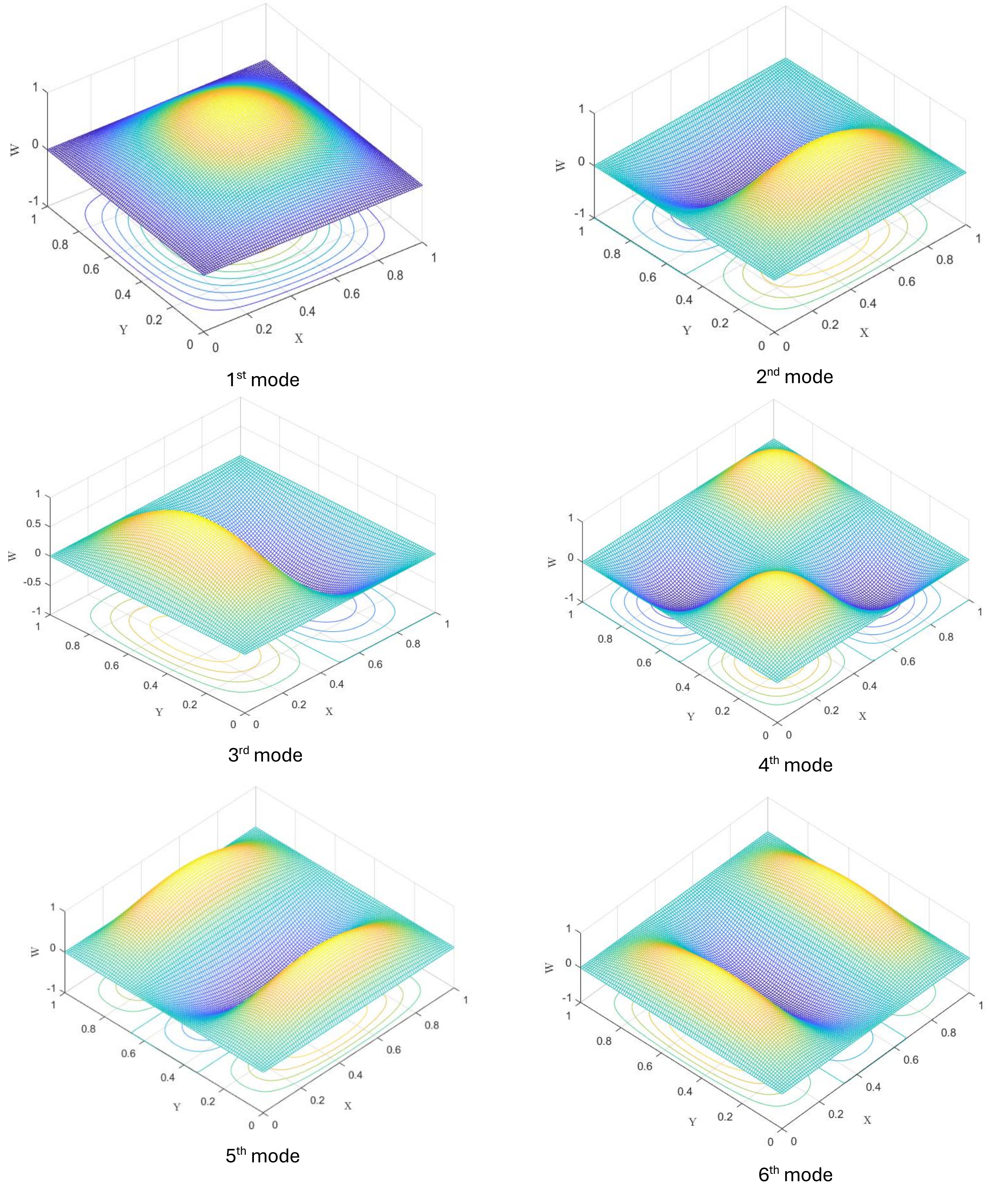

Figure 2. The first six mode shapes of rectangular thin plate with simply support boundary condition

Table 1. six natural frequencies of rectangular thin plate with simply support boundary condition

Natural frequency	Mode number					
	1(1,1)	2(1,2)	3(2,1)	4(2,2)	5(1,3)	6(3,1)
Precent	9.83	24.59	24.59	39.34	49.17	49.17
Yuan Du et al [1]	9.77	24.43	24.43	39.08	48.86	48.86

3. Sound power radiated from rectangular plate

The sound power radiation characteristics of simply supported rectangular thin plates are studied in this section. Using the Rayleigh surface integral where each element area on the rectangular plate is regarded as a simple point source of an outgoing wave, the acoustic pressure radiated from a vibrating rectangular plate in an infinite baffle can be obtained. The sound pressure can be expressed by velocity with Rayleigh integration [8, 9]:

$$p(r_n) = \frac{j\omega\rho_0}{2\pi} \iint_S v(r_n) \frac{e^{(-jkR)}}{R} dS \quad (13)$$

where $R = \sqrt{(x-x_0)^2 + (y-y_0)^2 + z^2}$ is the distance between any point r on the rectangular plate and observation point, r_n . According to the definition of sound intensity, the sound intensity of the observation point can be expressed as follows:

$$I(r_n) = \frac{1}{2} Re[p(r_n) v * (r_n)] \quad (14)$$

Then, $*$ stands for conjugate, the radiated sound power is:

$$W = \frac{1}{2} Re[\iint_S p(r_n) v * (r_n) dS] \quad (15)$$

The flat plate is divided into N units with equal area

and as mentioned, each discrete element of structure can be seen as point sound source, and the sound power is expressed as:

$$W = \frac{\omega\rho_0}{4\pi} \sum_{m=1}^{N} \sum_{n=1}^{N} v_m \frac{sin(kR)}{R} v_n * \Delta S.\Delta S \quad (16)$$

where are the velocities of unit Formula (18) is expressed as a matrix:

$$W = V^H R V \quad (17)$$

Where the (m, n) unit of matrix R is:

$$R_{mn} = \frac{\omega^2 \rho_0 (\Delta S)^2}{4\pi c_0} \frac{sin(kr_{mn})}{kr_{mn}} \quad (18)$$

If m=n, r=0, the matrix R is a real symmetric positive definite matrix.

Finally, acoustic radiated power level of the rectangular thin plate is expressed as below:

$$L_{W_p} = \frac{V^H R V}{W_0} \quad (19)$$

Where $W_0 = 10^{-12}(w)$ is the reference sound power.

4. Numerical simulation and discussion

The effect of thickness and location of point force on acoustic radiation characteristics are discussed in this section. Three different location of point force, and three different thickness is used for numerical simulations. The properties of rectangular plate used in this section are length a = 1 m, breadth b = 1 m, density ρ = 7,850 kg/m3, Passion's ratio μ = 0.3, Young's modulus E=2.1e11 Pa and the speed of sound c = 346 m/s.

When simply supported rectangular plate is forced by a harmonic concentrated force at point (0.5, 0.5), the sound power level radiated from rectangular plates with different thickness, can be obtained according to Eq. (19). They are shown in Fig. 3

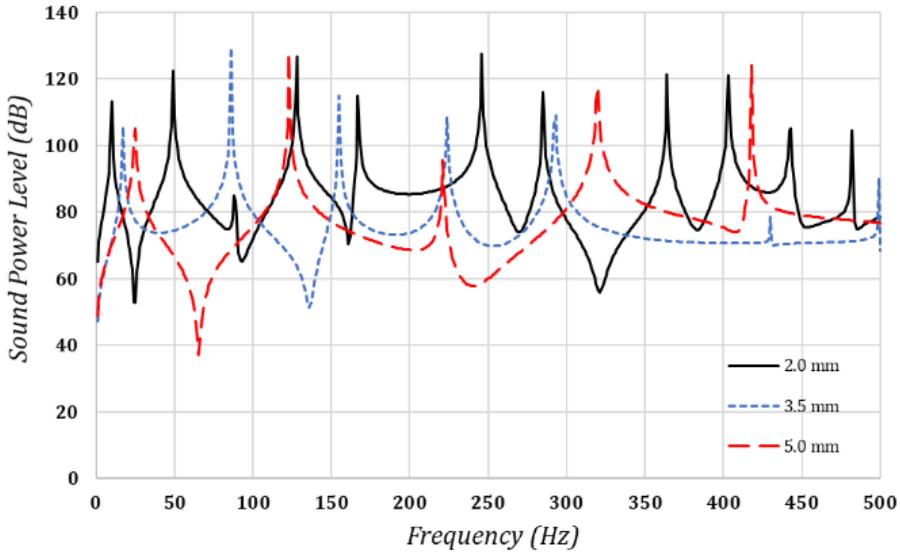

Figure 3. Sound power level of plate with different thickness

Figure 4 compares the sound power level of simply supported plate with the thickness of 5 mm, when the location of point force is different. From the data in Figure 4, we can see that the first peak value exists at same location of horizontal axis. In addition, with the point force moving to the center of plate, the number of peak decreases, whereas the peak value increases

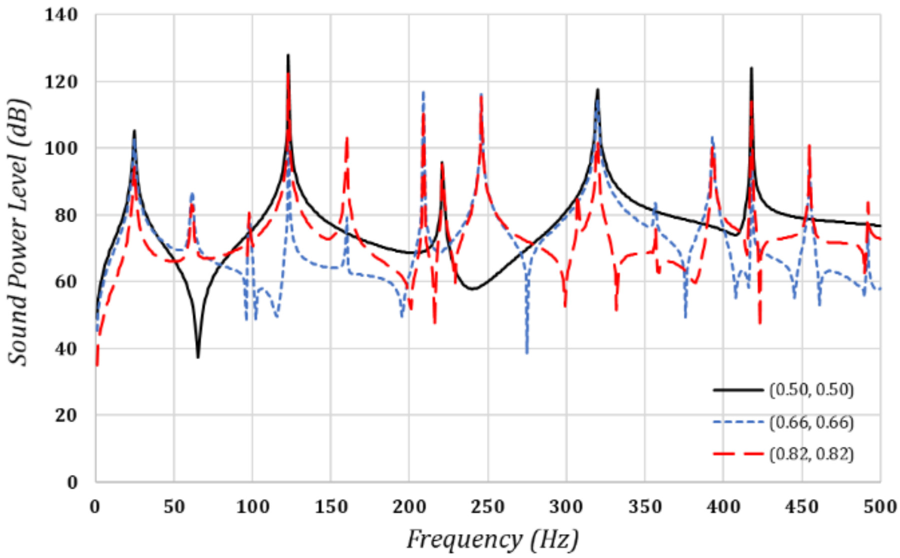

Figure 4. Sound power level of plate with different location of force point

5. Conclusions

In this paper, the vibro-acoustic response of a simply supported rectangular thin plate under different thickness and different point force location is established.

The study has identified with the increase of the plate thickness, sound radiation power curves will move to the low-frequency domain, which is caused by the structural characteristics. In order to obtain better low frequency sound transmission loss performance, it can be satisfied by increasing the thickness appropriately.

The research has also shown that when the point force moves to the center of four edges simply supported plate, the number of peak decreases, whereas the peak value increases.

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