Iranian Society of Acoustics and Vibration
The 14th ISAV2024 International Conference on Acoustics and Vibration
11-12 Dec 2024 Karaj - Iran
Kharazmi University

Design and optimization of a pre-stressed piezoelectric actuator based on compliant arc-beam mechanism

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Abstract

Over recent years, the piezoelectric materials are of interest due to their fast response, high precision, high output loads and compact sizes. On the other hand, due to the high sensitivity of piezoelectric materials to tensile and shear stresses, it is necessary to apply preload on these materials using. In this paper, a novel pre-stressed piezoelectric actuator based on compliant arc-beam mechanisms is designed and optimized. Dynamic governing equations based on spring-mass model are obtained using Hamilton's principle. To maximize the fundamental natural frequency and minimize the mass of the frame, the geometric parameters of the frame are optimized by multi-objective genetic algorithm (MOGA) which is conducted by multi-disciplinary MODFRONTIER software. Using ANSYS finite element software, a model of piezoelectric actuator is created to calculate stresses and natural frequencies. Then, by solving analytically the governing equations and using optimal geometrical parameters, the axial natural frequencies and the effect of harmonic accelerations and excitation voltage of piezoelectric stack on each other over a frequency range are investigated.

Keywords: Pre-stressed actuator; Piezoelectric; Compliant mechanism; Optimization.

1. Introduction

Nowadays, with the advancement of technology, the use of miniaturized systems, i.e., micro/nano systems, in many industries need precise, stable and fast functionality in the micro/nanometer scale such as biomedical, aerospace, microelectronics, optical equipment and micro positioning systems are of great importance. The smart actuators, referring to the response of such equipment to external stimulations such as voltage, are one of the main devices used as miniaturized systems to fulfil mentioned tasks [1].

Among various types of advanced materials, piezoelectric materials are of significant consideration due to their features such as quick response, high accuracy and precision, high output force, high stiffness, small dimensions and biological compatibility [2]. Despite the advantages that piezoelectric materials have, but due to small movement and very high sensitivity to tensile and shear stresses, the direct use of such smart materials in practical applications is rarely done. Thus, the development of piezo actuators has been performed to increase the amount of movement and prevent from creation of tensile or shear stresses, depending on application [3].

So far, numerous piezoelectric actuators have been designed and presented with different geometries and performances depending on the type of applications. Most of piezoelectric actuators were designated to amplified piezoelectric actuators (i.e., [4–7]) and less to pre-stressed ones. Mossi et. al. [8] studied the effects of different electrical fields and frequencies of external loads on displacement performance of two pre-stressed curved actuators. In one configuration, the piezoelectric stack sandwiched between a steel and aluminum sheets and in other one, the piezoelectric stack was between composite sheets. Huang et. al. [9] introduced a stack type piezoelectric actuator using Belleville springs so as to obtain an adjustable pre-stresses mechanism. Jiang and Cheng [10] presented a design method for a pre-stressed mechanism and analyzed both the mechanical and electrical aspects of the mechanism analytically and experimentally. In recent years, the pre-stressed piezoelectric actuators were designed for vibration isolation applications by Divijesh et.al [11].

In this paper, design, optimization and electro-mechanical analysis of a novel arc-beam compliant frame for piezoelectric actuator is performed. The frame is used to create a preload on the piezoelectric actuator. The dynamic governing equations of the piezoelectric actuator are derived based on spring-mass model using Hamilton's principle. To maximize the fundamental natural frequency and minimize the mass of the frame, the geometric parameters of the frame are optimized by multi-objective genetic algorithm (MOGA) is conducted by multi-disciplinary MODFRONTIER software. Then, by solving the governing equations and using optimal geometrical parameters, the effects of harmonic accelerations and excitation voltage on the stroke, the generated force, the current and the power consumption of piezoelectric stack are investigated.

2. Analytical model

Fig.1 shows a compliant arc-beam piezoelectric actuator, containing the frame and piezoelectric stack material. The total length (L), length of piezo stack (L_p), thickness (t), width (b), radius (R) and arc angle (θ) of arc-beam are depicted in Fig. 1.

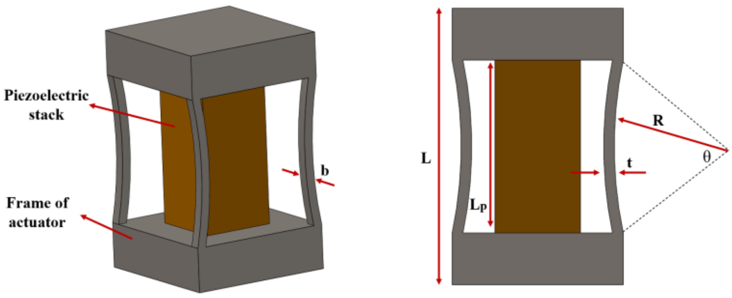

Figure 1. Pre-stressed compliant arc-beam piezoelectric actuator and geometrical parameters

To derive the dynamic governing equations, the piezoelectric actuator is modelled as two parallel springs with stiffness coefficients k_p for piezoelectric material and k_f for compliant arc-beam frame (Fig. 2). The equivalent stiffness of the presented actuator is equal to:

$$k_{eq} = k_f + k_p \quad (1)$$

where the stiffness coefficient of the frame (k_f) is calculated using finite element method (more details are expressed in section 3).

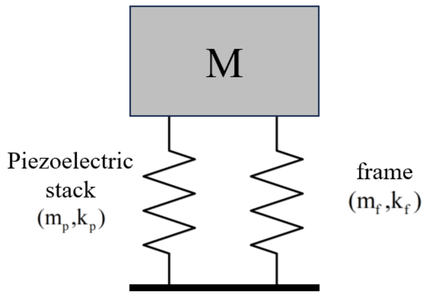

Figure 2. Spring-mass model of presented piezoelectric actuator

It should be noted that in piezoelectric materials, the performance is defined usually in terms of maximum stroke (Δ), the maximum deflection when the piezoelectric material is free, and blocking force (F_b), the maximum produced load by piezoelectric material when the excited voltage is at maximum (V_{max}) and the motion is completely blocked. In general, the relation between the produced load, deformation and voltage of piezoelectric materials is defined as follows [1]:

$$F_p = NV - k_p d \tag{2}$$

where F_p is produced load, N is equal to F_b / V_{max} and d is deflection of piezoelectric material.

The Hamilton's principle is used to derive dynamic governing equations, as follows [12]:

$$\delta I = \int_{t_1}^{t_2} \left[\delta T - \delta \pi - \delta W_{ext} \right] = 0 \tag{3}$$

where T is the kinetic energy, π is the strain energy function and W_{ext} V is the potential of external forces whose variations are defined as follows:

$$\begin{aligned} T &= \frac{1}{2} m_{ext} \dot{q}^2 + \frac{1}{2} \int_0^{l_f} \rho_f A_f \dot{q}^2 dx + \frac{1}{2} \int_0^{l_p} \rho_p A_p \dot{q}^2 dx \\ \pi &= \frac{1}{2} k_f \left(q + e \right)^2 \\ \delta W_{ext} &= -\left(F_{ext} + F_p \right) \delta q \end{aligned} \tag{4}$$

where M is external mass, ρ_f is density of frame, ρ_p is density of piezoelectric material, A_f is cross-section area of frame and A_p is cross-section area of piezoelectric material. Moreover, F_{ext} is external forced applied on the frame due to external mass (m_{ext}) and F_p is the force induced by piezoelectric material. In order to apply pre-stress load on piezoelectric material to avoid any failure, it is assumed that the length of the inner part of the frame is shorter than the piezoelectric length by amount of e. Also, q is the generalized coordinate along x-direction. In Eq. (4), the effect of frame mass and piezoelectric material on dynamic behaviour of actuator, are assumed as distributed mass springs, to increase accuracy. Hence, the inertia terms in the kinetic energy function is entered as integral forms. Assuming that the velocity changes along the springs are linear, as a result, the effective mass of the frame and piezoelectric material is equal to 1/3 of their total mass (i.e., $m_{p,eff} = m_p / 3$ and $m_{f,eff} = m_f / 3$ where $m_{p,eff}$ and $m_{f,eff}$ are effective masses).

integrating part by part, the dynamic governing equation of presented actuator is obtained as Eq. (5):

$$M\ddot{q} + Kq = F_{ext} + NV - k_f e \tag{5}$$

where $M = m_{ext} + m_{p,eff} + m_{f,eff}$ and $K = k_f + k_p$.

If it is assumed that the excitation voltage includes two parts of offset and dynamic voltage (i.e. $V = V_{off} + V_{dyn}$) and external forces comprises static and dynamic parts (i.e. $F_{ext} = F_{stc} + F_{dyn}$) then the general form of the equation will be as follows:

$$M\ddot{q} + Kq = F_{stc} + F_{dyn} + N\left(V_{off} + V_{dyn}\right) - k_f e \tag{6}$$

To obtain pre-stress load applied to the actuator, the dynamic terms, i.e., $\ddot{q}$ and V_{dyn}, must be set to zero in Eq. (6):

$$q_{pre} = \frac{F_{stc} + NV_{off} - k_f e}{K}. \tag{7}$$

The axial natural frequency ω_n of presented actuator is obtained as $\omega_n = \sqrt{K/M}$. If it is assumed that the q' is the displacement of actuator around the equilibrium point, which is defined as $q' = q - q_{pre}$, and the dynamic voltage is changed harmonically, i.e. $V_{dyn} = V_a \sin\left(\omega t\right)$ where V_a is the amplitude and ω is the frequency of applied voltage, then the displacement and acceleration of the actuator can be considered as follows:

$$q' = A\sin\left(\omega t\right) \quad \Rightarrow \quad \ddot{q}' = -\omega^2 A\sin\left(\omega t\right) \tag{8}$$

where A is the amplitude of harmonic vibration of actuator. By substitution of Eqs. (7) and (8) into Eq. (6), the amplitude and acceleration of actuator are as follows:

$$q'_{\max} = A = \frac{F_{dyn} + NV_a}{M\omega_n^2\left(1 - r^2\right)} \tag{9}$$

$$\ddot{q}'_{\max} = -\omega^2 A = \frac{F_{dyn} + NV_a}{M\omega_n^2\left(1 - r^2\right)} r^2 \tag{10}$$

where $r = \omega/\omega_n$. Eqs. (9) and (10) are the maximum displacement and acceleration amplitude of pre-stressed compliance arc-beam piezoelectric actuator under harmonic voltage and dynamic force.

3. Finite element simulation

To validate the analytical solution and calculate stiffness coefficient of actuator frame, a 3D solid model is created in ANSYS Workbench software. The model contains 5823 tetrahedrons elements with average size element 0.8 mm (Fig. 3). To calculate the stiffness coefficient of actuator frame, the static structural solver is used. In this case, the surface of the lower plate is fixed and the surface of the upper plate is subjected to certain displacement, while the piezoelectric material is not considered in this case. Then, the value of the reaction forces created in the structure is extracted and the stiffness coefficient of the actuator frame is obtained by dividing the reaction force by the displacement. In other case, to calculate the stresses posed on the actuator frame, like previous case, static structure solver of ANSYS Workbench software is used. In this case, it is assumed that the piezoelectric material applies the most possible force, i.e. blocking force where in this study is 900 N, on actuator frame. for modal analysis of the actuator, modal solver is utilized to obtain fundamental natural frequency and its equivalent mode shape. For both the second and third models (i.e. stress and modal analyses), the bottom of the actuator frame is fixed.

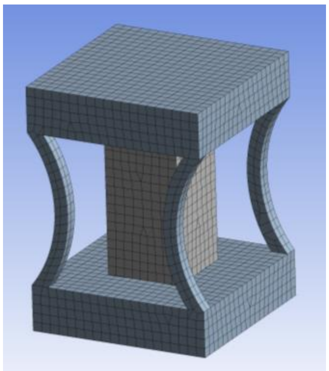

Figure 3. Meshed model of compliant arc-beam pre-stressed piezoelectric actuator

The core material of the frame is Titanium, with Young's modulus (E_f) 107 GPa, Poisson's ratio (ν_f) 0.35 and density (ρ_f) 4405 kg/m^3, and that of piezoelectric material is PZH-5H, with Young's modulus (E_p) 43.2 GPa, Poisson's ratio (ν_p) 0.3, density (ρ_p) 7850 kg/m^3 and stiffness coefficient (k_p) 108 N/mm.

4. Topology optimization

In this section, the procedure and numerical implementation on topology optimization of the compliant arc-beam actuator is described. The optimization is performed based on Multi-Objective Algorithm (MOGA) [13] using Modefrontier, a multidisciplinary optimization software. First, the actuator is modelled in one of the Computer Aided Design (CAD) software based on design variables and then, the model is imported into the ANSYS finite element software to perform static and modal analysis. Moreover, a Java script is written to calculate stiffness coefficient of actuator frame. The flowchart of the optimization procedure is depicted in Fig. 4.

The goals of optimization are to maximize the natural frequency, to minimize the mass of the frame, to make the stress below the yield strength of frame and to reach the stiffness coefficient equal to approximately 1/10 of piezoelectric material. The independent variables for optimization are thickness (t), width (b) and radius of the frame. Also, allowable bounds of each parameter and constrain are presented in Table 1. The MOGA parameters for optimization are initial population of 20, mutation probability of 10% and crossover probability of 50%.

5. Results and discussion

In the first section, the results of optimizing the geometrical parameters of the presented actuator are shown. In the second section, for verification purpose, the obtained solutions are compared with finite element model results. The fundamental natural frequency of the presented actuator with an external mass with 100 gr compared with the results of 3D finite element model. In the third section, parametric study is conducted to investigate the effects of dynamic voltage and harmonic acceleration on each other.

Table 1. Allowable bounds of parameters and constrains in optimization

Parameter	Allowable values
Thickness of arc-beam of frame (t)	$0.2 \leq t \leq 3.4$ mm
width of arc-beam of frame (b)	$0.3 \leq b \leq 3.4$ mm
radius of arc-beam of frame (R)	$6 \leq R \leq 25$ mm
Stiffness coefficient of frame (k_f)	$10000 \leq k_f \leq 12500$ N/mm
Stress created in frame (σ)	$\sigma \leq S_y$

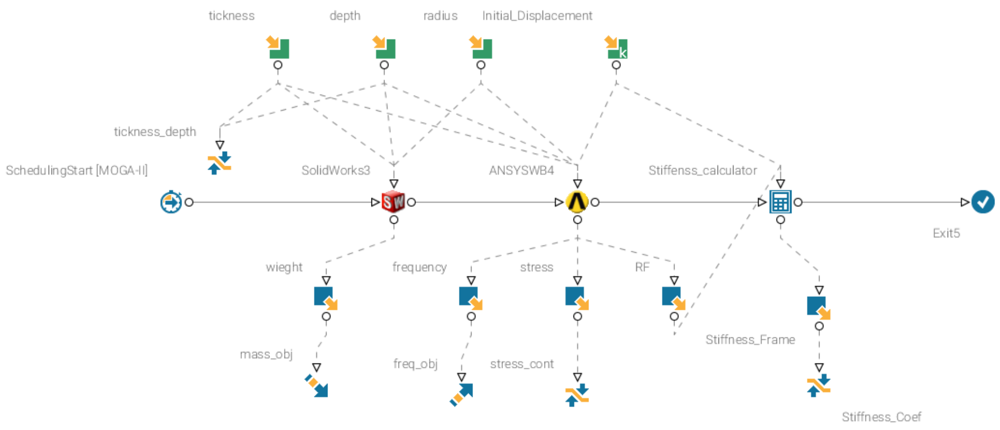

Figure 4. Optimization flowchart procedure in Modefrontier software

5.1 Optimization results

As it is mentioned, in this paper the optimized geometrical parameters of presented actuator obtained using MOGA. The set of parameters is finally chosen in a way that to have the closest stiffness to the desired one (i.e. 1/10 of piezoelectric stack stiffness). The optimized parameters, constrains and objective functions obtained as follows:

$$b = 0.7\,mm, \qquad t = 0.62\,mm, \qquad R = 24.1\deg$$
$$K_f = 11\,N/mm, \qquad \sigma = 228\,MPa, \qquad m = 8.75\,g, \qquad f = 29.5\,kHz$$

In Fig. 5, the initial and optimized geometry of frame is depicted

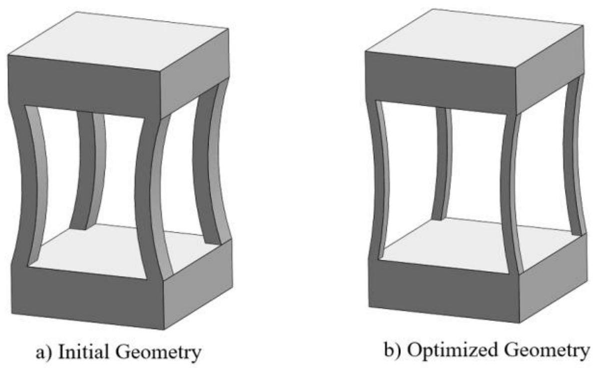

Figure 5. a) initial and b) optimized geometry of the frame

5.2 Validation

For verification of analytical model, the axial fundamental natural frequencies of actuator are compared with the results of ANSYS finite element codes for various external mass, presented in Table 2. It is seen that the axial natural frequencies obtained by presented analytical solutions are in an excellent with the ones obtained by optimized ANSYS finite element codes. The results show that with the increase of the external mass, the natural frequency is decreased, which is due to the overall increase of the mass of the system; While the stiffness of the system is not changed much.

5.3 Parametric study

Here, numerical results are presented to study the effects of dynamic voltage and harmonic acceleration on each other over the frequency range of 0 to 4000 Hz for presented actuator with optimized geometrical parameters. The external mass has a 100 gr mass and the lower surface of the frame is fixed.

Table 2. Comparison of fundamental axial natural frequencies of analytical model with optimized ANSYS finite element codes for various external masses

Mass (g)	Fundamental natural frequency (kHz)		Error (%)
	Analytical model	ANSYS (optimized)	
0	*26.15*	*27.93*	*-6.4*
40	*7.89*	*7.88*	*+0.1*
80	*5.71*	*5.65*	*+1.1*
120	*4.70*	*4.63*	*+1.5*
160	*4.08*	*4.02*	*+1.5*

In Fig. 6, the maximum amplitude of dynamic voltage (V_P) needed to produce specific harmonic acceleration is shown. The results show that to increase the harmonic acceleration, the required dynamic voltage also is increased at all frequencies. But on the other hand, in a certain acceleration, the required dynamic voltage is decreased with increasing frequency in such a way that at high frequencies, with a slight increase in voltage, the harmonic acceleration value can be increased a lot.

In Fig. 7, the effect of dynamic voltage on harmonic acceleration of presented actuator is studied over the frequency range of 0 to 4000 Hz. As expected, by increasing the amplitude of dynamic voltage, harmonic acceleration also is increased exponentially and the slope of harmonic acceleration is faster at high excitation voltages. On the other hand, in a certain dynamic voltage, the acceleration values also are increased with the increase in frequency.

6. Conclusion

In this study, a novel pre-stressed piezoelectric actuator based on a compliant arc-beam mechanism is designed and analysed its dynamic characteristics. The dynamic governing equations are obtained using Hamilton's principle and solved analytically. The frame and piezoelectric material are Titanium and PZH-5H, respectively. To obtain optimized geometrical parameters of presented actuator in such a way that maximize the fundamental natural frequency and minimize the mass of the frame, multi-objective genetic algorithm (MOGA) is used. By increasing frequency from 0 to 4000 Hz, due to approaching to the resonance, the desired excitation voltage and dynamic acceleration are astonishingly reduced and increased, respectively. The analytical results are in good agreement with numerical ones. By increasing dynamic acceleration from 1g to 12g, the excitation voltage of the actuator is decreased with closing to the resonance frequency. On the other hand, by increasing excitation voltage from 30V to 180V, the dynamic acceleration of the actuator is increased during the entire frequency range.

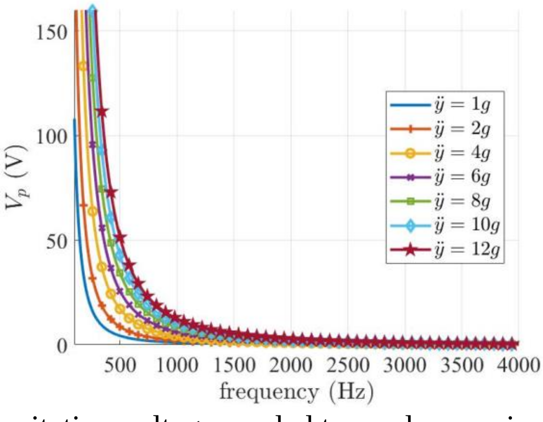

Figure 6. The excitation voltage needed to produce various harmonic acceleration

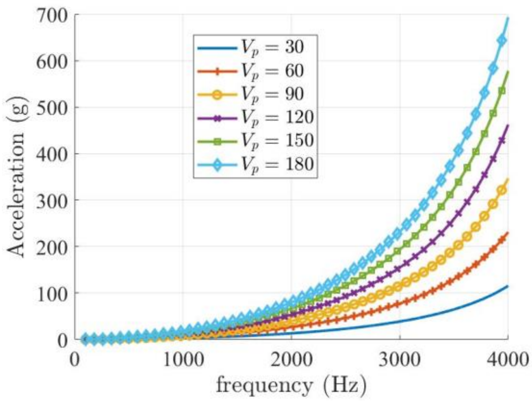

Figure 7. The effect of various excitation voltage on harmonic acceleration

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